Stress–Optic Relation and Photo-Viscoplasticity for Plastically Deforming Polymer Solids

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SYNOPSIS

Photo-viscoplasticity for finding the stress distribution in a plastically deforming polymer solid is discussed by using the deduced stress-optic relation. As an example, a practical application is shown for tension of the strip having semicircular notches in the plane stress state. © 1993 John Wiley & Sons, Inc.

INTRODUCTION

Deformation and stress states of plastically deforming polymer solids depend on the deformation history or the process leading to a given instant as well as the state at that instant. Accordingly, the plastic deformation is characterized by the relation between time rates of strain and stress. Moreover, the birefringent effect for the plastic deformation should be characterized by the time rate of isochromatic fringe.

Up to the present time, many experimental studies of birefringence for polymers have been reported.[1–6] In this article, photo-viscoplasticity for finding the stress distribution in a plastically deforming polymer solid is discussed by using the deduced optical relation. The deduced relation is differentiated with respect to time, and the stress–optic relation is obtained by using the constitutive equation for the plastically deforming polymer solid. Moreover, as an example of the stress analysis, a practical application is shown for tension of the strip having semicircular notches in the plane stress state.

STRESS–OPTIC RELATION OF PLASTICALLY DEFORMING POLYMER SOLIDS

For plastically deforming polymer solids used, the strain of the polymer solid is proposed to consist of the Hooken elastic strain E_e, the plastic strain E_p, and the creep strain E_c. Here, the inelastic strain is defined as $E_i = E_p + E_c$. The inelastic strain rate in the plane stress state is approximately expressed as shown in eq. (6) of the previous article[1] as follows

$$\begin{aligned}\Delta\dot{E}_i &= \Delta\dot{E}_p + \Delta\dot{E}_c \\ &= \frac{2n+1}{2G}\left(\frac{\Delta\sigma}{\sqrt{3}k}\right)^{2n}\Delta\dot{\sigma} \\ &\quad + B(t+s)^{\alpha}\exp\left[\left(\frac{b}{\sqrt{3}}\right)\Delta\sigma\right]\Delta\sigma, \quad (1)\end{aligned}$$

where the dot over ΔE represents differentiation with respect to time, $\Delta\sigma = \sigma_1 - \sigma_2$ and $\Delta E = E_1 - E_2$ are the differences of principal stresses and of principal strains, respectively. G, k, n, B, s, α, and b are material constants, and t denotes time.

The deduced optical relation of the first-order approximation is expressed as[1]

$$N = A(\tilde{\eta}_1 - \tilde{\eta}_2) = C_1(\Delta\sigma) + C_2(\Delta E_i), \quad (2)$$

where $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are the principal values of the index tensor in the dielectric field. C_1 and C_2 are optical constants. The time rate of eq. (2) becomes as

$$\dot{N} = C_1(\Delta\dot{\sigma}) + C_2(\Delta\dot{E}_i). \quad (3)$$

Equations (1) and (2) were verified experimentally to be effective for the cellulose nitrate at 65°C for several stress rates $\Delta\dot{\sigma}$.[1,7]

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By introducing eq. (1) into eq. (3), the stress–optic relation is obtained as

$$\dot{N} = C_1(\Delta\dot{\sigma}) + C_2\left\{\frac{2n+1}{2G}\left(\frac{\Delta\sigma}{\sqrt{3}k}\right)^{2n}\Delta\dot{\sigma} + B(t+s)^{\alpha}\exp\left[\left(\frac{b}{\sqrt{3}}\right)\Delta\sigma\right]\Delta\sigma\right\}. \quad (4)$$

DETERMINATION OF MATERIAL CONSTANTS

The material constants in eq. (1) can be determined experimentally by using the test results in creep and for proportional loading shown in the previous article.[1] In the case of the creep test $\Delta\sigma = c$ (const), or $\Delta\dot{\sigma} = 0$,

$$\Delta\dot{E}_i = B(t+s)^{\alpha}\exp[bc/\sqrt{3}]c \quad (5)$$

is obtained from eq. (1), and taking logarithms of both sides, the following form is obtained

$$\log(\Delta\dot{E}_i/c) = \log B + \alpha\log(t+s) + 0.434bc/\sqrt{3}.$$

As the values of $\Delta\dot{E}$ and t are known from the experimental results, the values of B, α, s, and b are approximately determined by the experiments for more than two different values of c.

For the case of proportional loading $\Delta\dot{\sigma} = d$ (const), or $\Delta\sigma = dt$, the following equation is obtained from eq. (1)

$$\Delta\dot{E}_i = B(t+s)^{\alpha}\exp[b(\Delta\sigma/\sqrt{3})]\Delta\sigma + \frac{2n+1}{2G}\left(\frac{\Delta\sigma}{\sqrt{3}k}\right)^{2n}d. \quad (6)$$

Unknown values in eq. (6) are G, k, and n. We can determine G in the range of elastic strain by using the elastic relation $\Delta E_e = \Delta\sigma/(2G)$ in eq. (5) of the previous article.[1] By denoting the known terms in eq. (6) with $A(t)$ as

$$A(t) = \Delta\dot{E}_i - B(t+s)^{\alpha}\exp[b(\Delta\sigma/\sqrt{3})]\Delta\sigma,$$

eq. (6) becomes

$$A(t) = \frac{2n+1}{2G}\left(\frac{\Delta\sigma}{\sqrt{3}k}\right)^{2n}d. \quad (7)$$

Taking logarithms of both sides of eq. (7)

$$\log A(t) = (2n)\log\Delta\sigma + \log(2n+1)d - \log 2G(\sqrt{3}k)^{2n}.$$

As the values of G, $A(t)$, and $\Delta\sigma$ are known, the values of k and n can be found. The values of material constants are obtained as $G = 31.5$ (kg/mm^2), $k = 0.9$ (kg/mm^2), $B = 7.7 \times 10^{-5}$ (kg/mm^2)$^{-1}$ (min)$^{-(\alpha+1)}$, $s = 1.0$ (min), $\alpha = -0.35$, $n = 1.8$, and $b = 6.5$ (kg/mm^2)$^{-1}$. The optical constants in eq. (3) were obtained as $C_1 = 0.23$ (mm/kg) and $C_2 = 1.7$ (mm)$^{-1}$ in the previous article.[1]

APPLICATION OF STRESS–OPTIC RELATION AND PHOTO-VISCOPLASTICITY

In actual plastically deforming polymer solids subjected to nonsteady inelastic as well as steady inelastic deformations, the relation between $N(t)$ and time t at each element of specimen is obtained from the isochromatic fringe pattern appearing in it; the relation $\dot{N}(t)$ and t is obtained from the gradient. In the calculation of $\Delta\sigma(t)$, time is subdivided into small intervals, $t_0 = 0 \sim t_1$, $t_1 \sim t_2, \ldots, t_{m-1} \sim t_m, \ldots, t$, in which the magnitude of the $\Delta\dot{\sigma}(t)$ is considered as constant.

Case 1

The case of $\Delta\sigma(t_0) = 0$ at $t = t_0 = 0$. As $\Delta\dot{E}_i(t_0) = 0$ is obtained from eq. (1) for $\Delta\sigma(t_0) = 0$ at $t = t_0 = 0$, the stress–optic relation (4) becomes

$$\dot{N}(t_0) = C_1\{\Delta\dot{\sigma}(t_0)\} + C_2\{0\}.$$

Therefore, the value of $\Delta\dot{\sigma}(t_0)$ is found from the value of $\dot{N}(t_0)$ obtained by the experiment as follows

$$\Delta\dot{\sigma}(t_0) = \frac{\dot{N}(t_0)}{C_1},$$

and the value of $\Delta\sigma(t_1)$ is found by the following relation

$$\Delta\sigma(t_1) = \Delta\dot{\sigma}(t_0)(t_1 - t_0).$$

In the period $t_1 \sim t_2$, eq. (4) at $t = t_1$ becomes

$$\dot{N}(t_1) = C_1\{\Delta\dot{\sigma}(t_1)\} + C_2\left\{\frac{2n+1}{2G}\left(\frac{\Delta\sigma(t_1)}{\sqrt{3}k}\right)^{2n}\Delta\dot{\sigma}(t_1)\right.$$

$$+B(t_1+s)^{\alpha}\exp\left[\left(\frac{b}{\sqrt{3}}\right)\Delta\sigma(t_1)\right]\Delta\sigma(t_1)\Bigg\}$$

$$= L_1\Delta\dot{\sigma}(t_1) + M_1,$$

where L_1 and M_1 are known values for the each element. Therefore, the value of $\Delta\dot{\sigma}(t_1)$ is found from the value of $\dot{N}(t_1)$ obtained by the experiment as follows

$$\Delta\dot{\sigma}(t_1) = \frac{\dot{N}(t_1) - M_1}{L_1}$$

and the value of $\Delta\sigma(t_2)$ is found by the following relation

$$\Delta\sigma(t_2) = \Delta\sigma(t_1) + \Delta\dot{\sigma}(t_1)(t_2 - t_1).$$

In the same manner, the value of $\Delta\sigma(t)$ in every instant for each element is obtained from the corresponding value of $\dot{N}(t)$ obtained by experiment.

Case 2

The case of $\Delta\sigma(t_0) \neq 0$ at $t = t_0 = 0$. This corresponds to the case in which a load is applied instantly at $t = t_0$, and the load varies with lapse of time. In the period $t_0 = 0 \sim t_1$, as the fringe order $N(t_0)$ is found from the experimental result, the value of $\Delta\sigma(t_0)$ is easily decided from the experimental relation between $\Delta\sigma$ and N obtained from calibration tests. Equation (4) at $t = t_0 = 0$ becomes

$$\dot{N}(t_0) = C_1\{\Delta\dot{\sigma}(t_0)\} + C_2\left\{\frac{2n+1}{2G}\left(\frac{\Delta\sigma(t_0)}{\sqrt{3}k}\right)^{2n}\Delta\dot{\sigma}(t_0) + \cdots\right\}$$

$$= L_0\Delta\dot{\sigma}(t_0) + M_0.$$

Therefore, the value of $\Delta\dot{\sigma}(t_0)$ is found from the value of $\dot{N}(t_0)$ obtained by the experiment as follows

$$\Delta\dot{\sigma}(t_0) = \frac{\dot{N}(t_0) - M_0}{L_0}$$

and the value of $\Delta\sigma(t_1)$ is found by the following relation

$$\Delta\sigma(t_1) = \Delta\sigma(t_0) + \Delta\dot{\sigma}(t_0)(t_1 - t_0).$$

The procedure after that is the same as in Case 1.

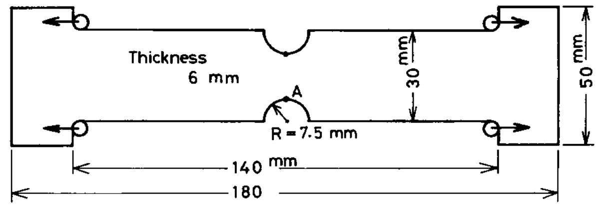

Figure 1 Dimensions of test specimen.

EXAMPLE OF STRESS ANALYSIS

As an example, the stress distribution in a cellulose nitrate strip having semicircular notches was analysed by using the photo-viscoplasticity mentioned in the previous section. The strip having semicircular notches shown in Figure 1 was made of a transparent cellulose nitrate plate 6-mm thick made of the same polymer solid as in the previous article,[1] and was applied to tensile load at 65°C. Let σ_m be the average stress over the minimum cross section of the specimen shown in Figure 1. The experiments were performed under the constant rates of $\dot{\sigma}_m = 0.01$ and $0.05\ \mathrm{kg/mm^2/min}$ (namely, the value of σ_m increases linearly with time). The isochromatic fringe patterns were recorded at suitable time intervals.

As an example of the results obtained, Figure 2 shows the relation between the fringe order per unit thickness $N(t)$ and time with solid curves for each value of $\dot{\sigma}_m$ at the bottom of the notch (point A in Fig. 1), and $\dot{N}(t)$ and time with dashed curves. Fig-

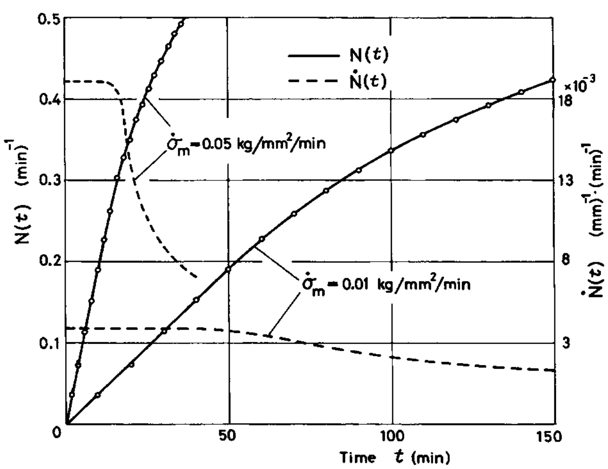

Figure 2 Relations between the fringe order N and time t, and $\dot{N}$ and t at the bottom of the notch for each value of $\dot{\sigma}_m$.

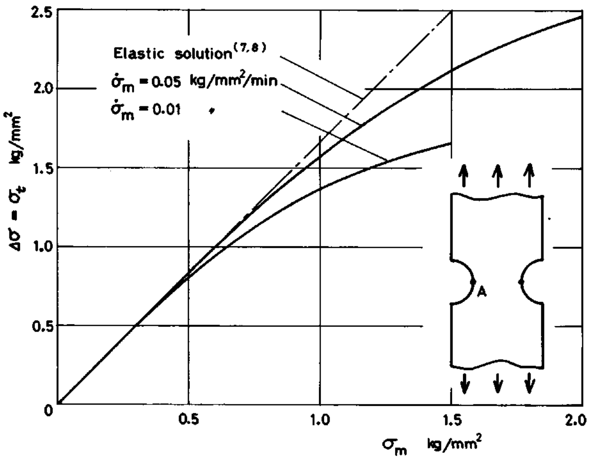

Figure 3 Relations between σ_t and σ_m for each value of $\dot{\sigma}_m$.

ure 3 shows the difference of principal stresses $\Delta\sigma$ at this point A for each value of $\dot{\sigma}_m$ calculated by the procedure mentioned in the previous section, where the principal stress normal to the boundary of the notch is equal to the zero and the tangential principal stress σ_t is equal to $\Delta\sigma$. The chain line in Figure 3 corresponds to the elastic result.[8,9] As shown in Figure 3, the values of σ_t for the plastically deforming polymer solid are affected by the stress rate or the stress history.

CONCLUSIONS

The deduced stress–optic relation is useful for finding the stress distribution for the plastically deforming polymer solid. Interesting results may be obtained for inelastic deformation by using the proposed photo-viscoplasticity.

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